Assume the symbolism $+\sqrt{j}=-1$, *j being an impossible quantity, the root of the equation* $+\sqrt{x}+1=0$.

Square (1), $1+10/y-3/y^2=4j$, $y^2(4j-1)-10y+3=0$, from which

$$x=\frac{10j\pm\sqrt{(7-3j)}}{4j-1}.$$

III. Solution by the PROPOSER.

Transpose and square.

$\therefore\ 4x^2-20x+25=x^2-7$ ......(B), an obvious quadratic.

Apply its roots, 4 and $\frac{8}{3}$, to the given (A); hence $2(4)+[-3]=8-3=5$; ......$=2x+\{-[\sqrt{(16-7)}]\}$..... (C); and

$$2(\tfrac{8}{3})+(-\tfrac{1}{3})=5\tfrac{1}{3}-\tfrac{1}{3}=5\ldots\ldots=2x+\{-[\sqrt{(\tfrac{64}{9}-\tfrac{63}{9})}]\}\ldots\ldots(D);$$

satisfy it. Could extracting $\sqrt{(x^2-7)}$ *positive* here *also* yield roots, then (A)'s dominant *quadratic* (B) is *bi*-quadratic, which is absurd.

Also solved by *P. S. BERG* and *CHAS. C. CROSS*.

GEOMETRY.

127. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

The equation to the plane through the extremities, (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3), of conjugate diameters of the ellipsoid,

$$\frac{x^2}{y^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1 \text{ is } \frac{x_1+x_2+x_3}{a^2}x+\frac{y_1+y_2+y_3}{b^2}y+\frac{z_1+z_2+z_3}{c^2}z=1.$$

Solution by the PROPOSER.

If $lx+my+nz=p$ ....(1) be the required plane, we should have

$$lx_1+my_1+nz_1=p\ldots\ldots(2),$$
$$lx_2+my_2+nz_2=p\ldots\ldots(3),$$
$$lx_3+my_3+nz_3=p\ldots\ldots(4).$$

Solving these for l/p, m/p, n/p, we have

$$l/p=\begin{vmatrix}1, & y_1, & z_1\\ 1, & y_2, & z_2\\ 1, & y_3, & z_3\end{vmatrix}\div\begin{vmatrix}x_1, & y_1, & z_1\\ x_2, & y_2, & z_2\\ x_3, & y_3, & z_3\end{vmatrix}\ldots\ldots(5).$$

$m/p=$etc., $n/p=$etc., ... (6).

Reducing (5), making use of

$$\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}+\frac{z_1^2}{c^2}=1=\frac{x_2^2}{a^2}+\ldots\ldots=\frac{x_3^2}{a^2}+\ldots\ldots(7),$$

and $x_1y_1+x_2y_2+x_3y_3=y_1z_1+\text{etc.},=z_1x_1+\text{etc.},=0\ldots\ldots(8)$,

$$l/p=\frac{x_1+y_1+z_1}{a^2},\ m/p=\text{etc.},\ n/p=\text{etc.},\ \ldots\ldots(9).$$

These must be put into (1).

Also solved by *G. B. M. ZERR*, *J. W. YOUNG*, *LON C. WALKER*, *J. SCHEFFER*, and *GEORGE LILLEY*.

128. Proposed by W. H. CARTER, Vice President and Professor of Mathematics, Centenary College, Jackson, La.

Given $F=\triangle^{n-1}\div(n-1)!.\triangle_1.\triangle_2\ldots\ldots\triangle_n$, where $\triangle$=the determinant $(a_1b_2c_3\ldots\ldots k_n)$ and $\triangle_1\triangle_2\ldots\ldots\triangle_n$ are the minors of the elements of the nth column; and a_m, b_m, $c_m\ldots\ldots$etc., $(m=1, 2, 3\ldots\ldots n)$ are the coefficients of n given equations containing $n-1$ variables. Show (1) that $n=3$, F=the area of a triangle, and (2) if $n=4$, F=the volume of the tetrahedron.

Solution by J. W. YOUNG, Student in Ohio State University, Columbus, O.

1. Let $n=3$. The points of intersection of the three lines represented by the given equations, are

$$x_1=-\frac{A_1}{C_1};\ x_2=-\frac{A_2}{C_2};\ x_3=-\frac{A_3}{C_3};$$

$$y_1=-\frac{B_1}{C_1};\ y_2=-\frac{B_2}{C_2};\ y_3=-\frac{B_3}{C_3};$$

where, by the usual notation, A_k equals the co-factor a_k, in the determinant $\triangle$.

The area of the triangle formed by these points is

$$\text{Area}=\tfrac{1}{2}\begin{vmatrix} x_1, & y_1, & 1 \\ x_2, & y_2, & 1 \\ x_3, & y_3, & 1 \end{vmatrix}=\tfrac{1}{2}\begin{vmatrix} -\frac{A_1}{C_1}, & -\frac{B_1}{C_1}, & 1 \\ -\frac{A_2}{C_2}, & -\frac{B_2}{C_2} & 1 \\ -\frac{A_3}{C_3}, & -\frac{B_3}{C_3} & 1 \end{vmatrix}=\tfrac{1}{2}\begin{vmatrix} A_1, & B_1, & C_1 \\ A_2, & B_2, & C_2 \\ A_3, & B_3, & C_3 \end{vmatrix}\div C_1C_2C_3$$

and this, by a well-known theorem in determinants,

$$=\tfrac{1}{2}\triangle^2\div C_1C_2C_3=F.$$

2. Let $n=4$. The intersections of the four planes given by the equations are found precisely as above.